DETERMINANTS

Exercises 3

Q(1). Evaluate (a) $\begin{vmatrix} 28 & 25 & 38 \\ 42 & 38 & 65 \\ 56 & 47 & 83 \end{vmatrix}$.

(b) $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$

Q(2). Prove that without expanding, that each of the following determinants vanishes. (a) $\begin{vmatrix} 1 & 15 & 14 & 4 \\ 12 & 6 & 7 & 9 \\ 8 & 10 & 11 & 5 \\ 13 & 3 & 2 & 16 \end{vmatrix} =$

(b) $\begin{vmatrix} 1 & a & a^2 & a^3+bcd \\ 1 & b & b^2 & b^3+cda \\ 1 & c & c^2 & c^3+dab \\ 1 & d & d^2 & d^3+abc \end{vmatrix} =$

Q(3). Show that $\begin{vmatrix} x^2+1 & xy & xz & xu \\ xy & y^2+1 & yz & yu \\ zx & zy & z^2+1 & zu \\ ux & uy & uz & u^2+1 \end{vmatrix} = x^2+y^2+z^2+u^2+1.$

Q(4). Solve the equation

(a) $\begin{vmatrix} a+x & b+x & c+x \\ b+x & c+x & a+x \\ c+x & b+x & a+x \end{vmatrix} = 0,$

(b) $\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 3x+5 & 5x+8 & 10x+17 \end{vmatrix} = -2(x+1)^2(x+2)$

Q(5). Prove, without expanding , that each of the following determinants vanishes,

(a) $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix}$

(b) $\begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ a-b & b-c & c-a \end{vmatrix}$

Q(6). Prove the followings:

$$\begin{vmatrix} a & b & c \\ l & m & n \\ p & q & r \end{vmatrix} = \begin{vmatrix} m & b & q \\ l & a & p \\ n & c & r \end{vmatrix} = \begin{vmatrix} l & m & n \\ p & b & r \\ a & b & c \end{vmatrix}$$

Q(7). $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

Q(8).

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+a & 1 & 1 \\ 1 & 1 & 1+a & 1 \\ 1 & 1 & 1 & 1+a \end{vmatrix} = a^4 + 4a^3$$

Q(9) . $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix}$. $= (b-a)(c-a)(c-b)$

Q(10).

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

Q(11). Factorize

$$\begin{vmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & d & d^2 & d^3 \end{vmatrix} = (b-a)(c-a)(d-a)(c-b)(d-b)(d-c)$$

Q(12). Show that :

$$\begin{pmatrix}(x-a)^2 & (y-a)^2 & (z-a)^2\\(x-b)^2 & (y-b)^2 & (z-b)^2\\(x-c)^2 & (y-c)^2 & (z-c)^2\end{pmatrix}=\begin{pmatrix}1 & -2a & a^2\\1 & -2b & b^2\\1 & -2c & c^2\end{pmatrix}\begin{pmatrix}x^2 & y^2 & z^2\\x & y & z\\1 & 1 & 1\end{pmatrix}$$

Q(13). Prove that

$$\begin{vmatrix}a^2+p & ab & ac & ad\\ab & b^2+p & bc & bd\\ac & bc & c^2+p & cd\\ad & bd & bc & d^2+p\end{vmatrix}=p^3(a^2+b^2+c^2+d^2+p)$$

$$=\underline{\underline{(1+a^2+b^2+c^2+d^2)}}p^3$$

Q(14). Let a_{ij} be an i-j element of determinant Dn of order nxn, define as follows

$$a_{ij} = a; \quad \text{if} \quad i<j$$

$$a_{ij} = x; \quad \text{if} \quad i=j$$

$$a_{ij} = -a; \quad \text{if} \quad i>j$$

$$D_n=\begin{vmatrix}x & a & a & \ldots\ldots & \ldots\ldots\ldots & a\\-a & x & a & a & \ldots & a\\-a & -a & x & a & \ldots. & a\\\ldots. & \ldots. & \ldots & \ldots & \ldots\ldots & \ldots\\-a & -a & -a & -a & x & a\\-a & -a & -a & -a & -a & x\end{vmatrix}, \quad \underline{\underline{D_n=\frac{1}{2}\left[(x-a)^n+(x+a)^n\right]}}$$

Q(15).

$$\begin{vmatrix} a_1+1 & a_1 & a_1 & .. & a_1 \\ a_2 & a_2+1 & a_2 & .. & a_2 \\ a_3 & a_3 & a_3+1 & .. & a_3 \\ .. & .. & .. & .. & .. \\ a_n & a_n & a_n & .. & a_n+1 \end{vmatrix}_n = \underline{\underline{(1+a_1+a_2..+a_n)}}$$

Q(16). Prove that

$$\begin{vmatrix} a & b & b & .... & b \\ b & a & b & .... & b \\ b & b & a & ... & b \\ .. & .... & .... & .... & .... \\ b & b & b & ..... & a \end{vmatrix} \qquad = \underline{\underline{(a-b)^{n-1}(a+(n-1)b)}}$$

Q(17). Show that

$$\begin{vmatrix} a & -b & -a & b \\ b & a & -b & -a \\ c & -d & c & -d \\ d & c & d & c \end{vmatrix} \qquad = \underline{\underline{4(a+b^2)(c^2+d^2)}}$$

Q(17). D_n is the nth order determinant

$$D_n = \begin{vmatrix} 1+x^2 & x & 0 & \dots & 0 \\ x & 1+x^2 & x & \dots & 0 \\ 0 & x & 1+x^2 & \dots & 0 \\ \dots & \dots & x & 1+x^2 & x \\ 0 & 0 & 0 & x & 1+x^2 \end{vmatrix}_n$$

$$\underline{\underline{D_n = (1+x^2)D_{n-1} - x^2 D_{n-2}}}$$

Q(18).

$$\begin{vmatrix} 0 & 1 & 2 & 3 & 4 & \dots & n-1 \\ 1 & 0 & 1 & 2 & 3 & \dots & n-2 \\ 2 & 1 & 0 & 1 & 2 & \dots & n-3 \\ 3 & 2 & 1 & 0 & 1 & \dots & n-4 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ n-2 & n-3 & n-4 & n-5 & \dots & \dots & 1 \\ n-1 & n-2 & n-3 & n-4 & & 1 & 0 \end{vmatrix}$$