3.2 Active filters

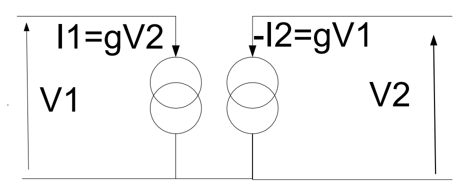
The filters that we have studied so far have all been passive filters – that is, they consisted only of passive components. The Butterworth and Chebyschev filters were both passive reactive filters and had only capacitors and inductors.

With the advent of LSI and VLSI, it became necessary to look at the possibility of implementing a filter circuit within an IC. It is possible to implement resistors and capacitors using film technology (even though implementing resistors is comparatively more difficult), but there is no way to construct an inductor. This necessitated the development of techniques to realise networks with inductors using only resistors and capacitors. Active circuits such as gyrators and negative impedance converters are used for this purpose.

NICs and gyrators are implemented using operational amplifiers. Operational amplifiers may also be used directly to implement transfer functions representing filter characteristics, using their high gaim, high input impedance and low output impedance characteristics. There are however limitations on their frequency spectrum, and filters operating at high frequencies, such as in communication circuits are still implemented using inductors.

3.2.1 Active filters using gyrators

A gyrator is a device that converts a load impedance in to an input impedance proportional to its inverse. Assuming a voltage controlled current source (VCCS), we can represent a gyrator as follows. [The constant of proportionality (g) between the voltage and the current is called its gyration ration.]



Representing this circuit with its (ABCD) parameters, we have:

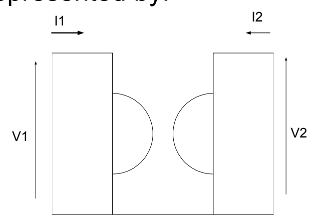
$$V_1 = AV_2 + BI_2$$
$$I_1 = CV_2 + DI_2$$

$$A = \left|\frac{V_1}{V_2}\right|_{I_2=0} = 0, \qquad B = \left|\frac{V_1}{I_2}\right|_{V_2=9} = -\frac{1}{g}, \qquad C = \left|\frac{I_1}{V_2}\right|_{I_2=0} = g, \qquad D = \left|\frac{I_1}{I_2}\right|_{V_2=0} = 0$$

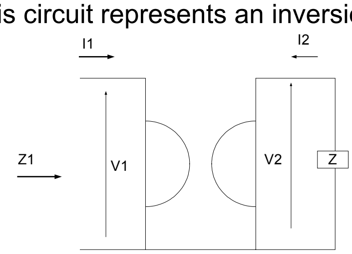
This gives:

$$V_1 = -g^{-1} I_2$$
$$I_1 = g V_2$$
$$or$$
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 0 & g^{-1} \\ g & 0 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Symbolically, a gyrator is represented by:

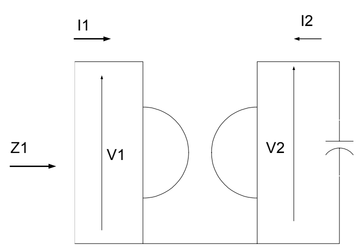


We will now show that this circuit represents an inversion of the impedance.



$$Z_1 = \frac{V_1}{I_1} = \frac{-g^{-1} I_2}{g V_2} = \frac{1}{-g^2 \dfrac{V}{I_2}} = \frac{1}{g^2 Z}$$

Example

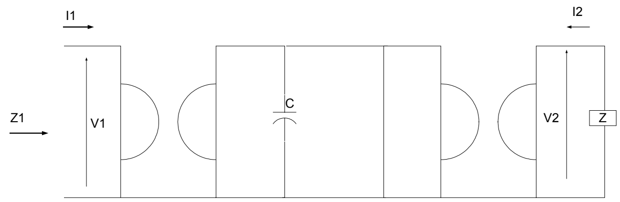


If we have a capacitance C at the output, then,

$$Z = \frac{1}{Cs}$$
$$Z_1 = \frac{1}{g^2 Z} = \frac{Cs}{g^2}$$

This is the same as the impedance of an inductance L = C/g^2.

To obtain the equivalence for a series inductance, we have to connect two gyrators in cascade:



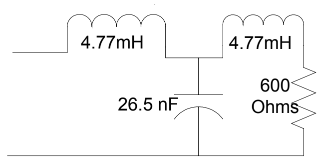
If the terminal impedance is Z, the impedance looking in at the intermediate port is $1/g^2Z$, as before. This now appears in parallel with the impedance of the capacitance C to give $\frac{1}{Cs+g^2Z}$

With this at the output port of the first gyrator, the input impedance is:

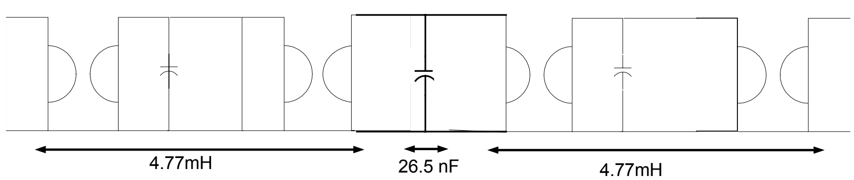
$$Z_1 = \frac{1}{g^2 \dfrac{1}{Cs+g^2Z}} = \frac{Cs+g^2Z}{g^2} = \frac{Cs}{g^2} + Z$$

This is equivalent to a series inductance of C/g^2, followed by the original impedance across the final output port.

Let us now see how we can implement a complete LC filter without the use of inductors. We will consider a circuit derived in an earlier lecture::

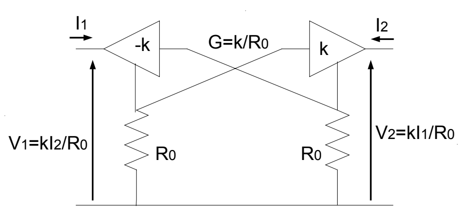


This may be realised as:



Realisation of the gyrator

The following circuit is a possible realisation of a gyrator using operational amplifiers. It approximates an ideal gyrator as $R_0 \to 0$.

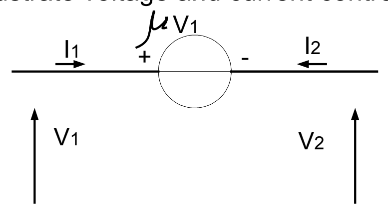


3.2.2 Active filters using NICs

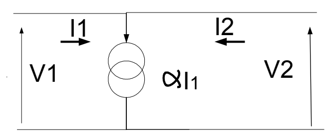
Another device used in the design of active filters is the negative impedance converter (NIC).

An ideal NIC is a two-port network, which converts a load impedance to an input impedance, which is proportional to the negative of the load impedance.

The following figures illustrate voltage and current controlled NICs



Voltage controlled NIC



Current controlled NIC

Their ABCD parameters are:

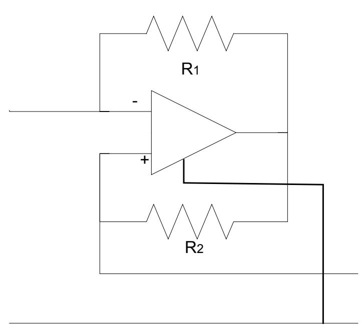
Voltage controlled NIC (VNIC):

$$\begin{bmatrix} \frac{1}{1-\mu} & 0 \\ 0 & 1 \end{bmatrix}$$

Current controlled NIC (INIC)

$$:\begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{1-\alpha} \end{bmatrix}$$

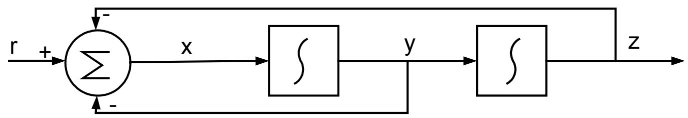
Realisation of an INIC (The ration k= $R_1/R_2 = 1/(\alpha-1)$)



3.2.3 The state-variable filter

We have earlier (in Chapter 2) seen how to obtain a state-space representation of a transfer function, and how such a system may be realised using a series of integrators and summers. This approach may be used to implement a filter using operational amplifiers. An operational amplifier with capacitive feedback functions as an integrator while resistive feedback produces a summer.

We will see how to connect two integrators and a summer to obtain a second order filter realisation.



We have the relationships:

$$x = r - y - z, \qquad y = \frac{x}{s}, \qquad z = \frac{y}{s}$$

Rearranging to get the transfer functions between r and the other variables:

$$x = r - \frac{x}{s} - \frac{x}{s^2}$$

$$r = x\left[1 + \frac{1}{s} + \frac{1}{s^2}\right] = x\left[\frac{s^2 + s + 1}{s^2}\right]$$

$$\frac{x}{r} = \frac{s^2}{s^2 + s + 1}$$

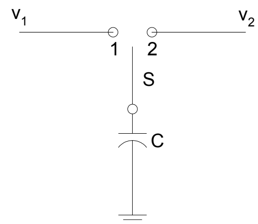
$$\frac{y}{r} = \frac{s}{s^2 + s + 1}$$

$$\frac{z}{r} = \frac{1}{s^2 + s + 1}$$

We see that the outputs x, y and z correspond to standard second order high-pass, band-pass and low-pass filter responses. The coefficients of the terms in s and s^2 may be adjusted simply by changing the gain and integrator time constants as desired. We have also seen before that any higher order transfer function may be broken down into a series of second order terms and possibly one first order term. This fact is used to implement higher order filters.

3.2.4 Switched capacitor filters

Even though it is possible to build in resistors into integrated circuits, they do present difficulties. It is advantageous if we can dispense with their use and use only capacitors. It is also difficult to achieve great accuracy in the construction of individual resistors and capacitors (typical accuracies are in the order of 30%) whereas accuracies of almost three orders of magnitude lower (about 0.05%) are possible if we only require ratios between capacitors, rather than their absolute values. These are the motivations for the development of switched capacitor filters. We will first try to understand how switched capacitors can simulate a resistor.



The figure shows a capacitor C connected to a switch S that has two positions 1 and 2, connected to two terminals maintained at voltages v_1 and v_2. For simplicity, we will assume that all connections are ideal and without resistance, so that the capacitor charges up instantly to the relevant voltage upon connection to either of the two terminals.

When S is at 1, the capacitor charges up to voltage v_1, so that the charge on the capacitor is Cv_1. If the switch is now thrown over to 2, the charge changes to Cv_2, that is, a charge of $C(v_1 - v_2)$ is delivered from 1 to 2 when the switch is thrown from 1 to 2, assuming that $v_1 > v_2$.

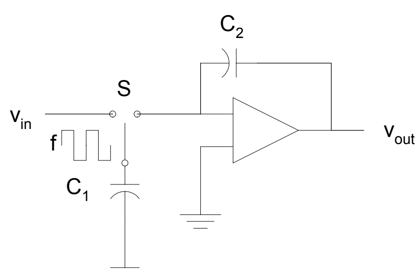
Let us now assume that the switch toggles between 1 and 2, at a frequency of f Hz. Then, the charge transfer per unit time (that is, the current flow) is

$$i = C(v_1 - v_2)\, f$$

Comparing this with the current flow through a resistor R connected between 1 and 2, we can write:

$$R = 1/Cf$$

We have a simulated resistor, whose value may be changed at will simply by changing the clock frequency! The other advantage we were looking for, that of only requiring ratios between capacitors rather than their absolute values, becomes apparent when we use this simulated resistor to construct an integrator using an operational amplifier. It is best to use ratios close to unity, for greater accuracy.



The above integrator has a natural frequency of $C_1 f / C_2$ or a time constant of $C_2 / C_1 f$.